Physical Models of an Elevator

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Abstract
An elevator is used as a simple system to model a few physical aspects. We will show simple kinematic models and we will consider energy consumption. These low level models are used to understand (physical) design considerations. Elsewhere we discuss higher level models, such as use cases and throughput, which complement these low level models.

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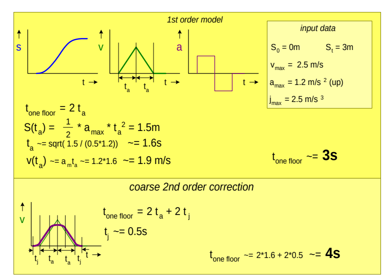


warning

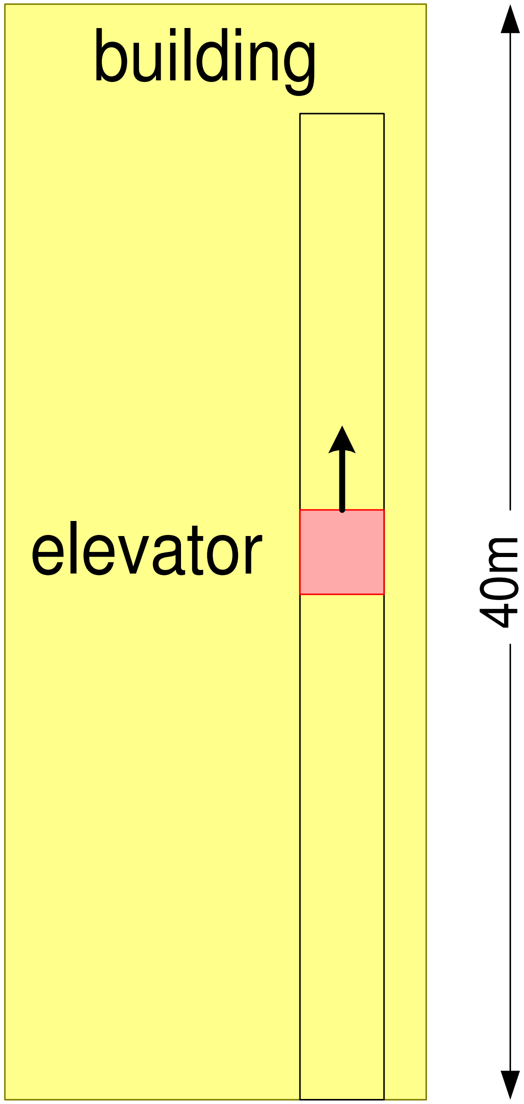
This presentation starts with a trivial problem.

Have patience!

Extensions to the trivial problem are used to illustrate many different modeling aspects.

Feedback on correctness and validity is appreciated

The Elevator in the Building



inhabitants want to reach
their destination fast and comfortable

building owner and *service operator*
have economic constraints:
space, cost, energy, ...

Elementary Kinematic Formulas

S_t = position at time t

v_t = velocity at time t

a_t = acceleration at time t

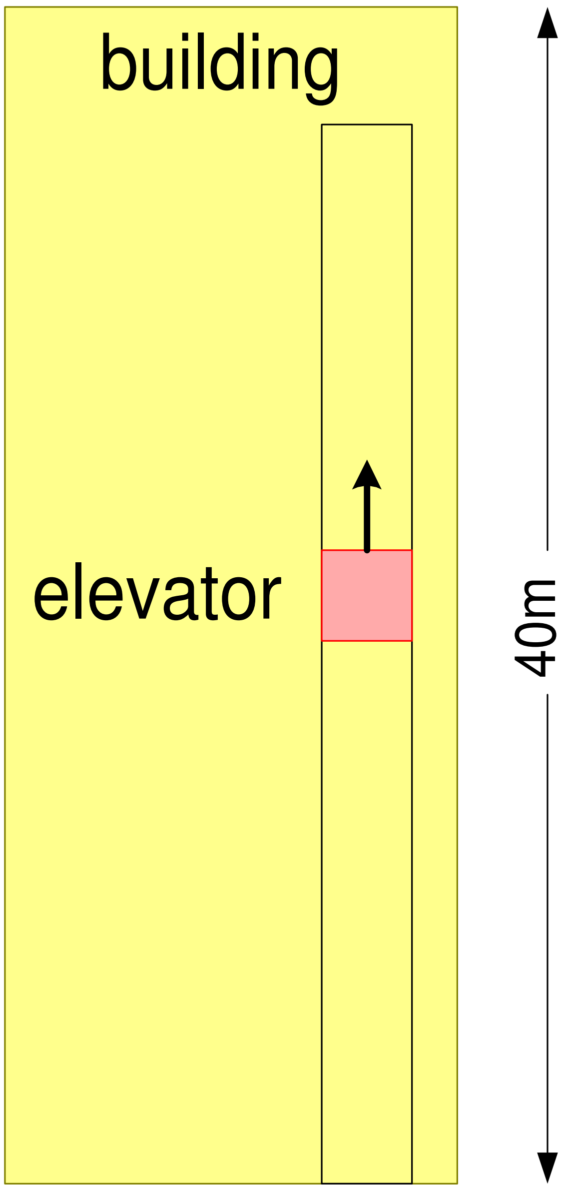
j_t = jerk at time t

$$v = \frac{dS}{dt} \qquad a = \frac{dv}{dt} \qquad j = \frac{da}{dt}$$

Position in case of uniform acceleration:

$$S_t = S_0 + v_0 t + \frac{1}{2} a_0 t^2$$

Initial Expectations



What values do you expect or prefer for these quantities? Why?

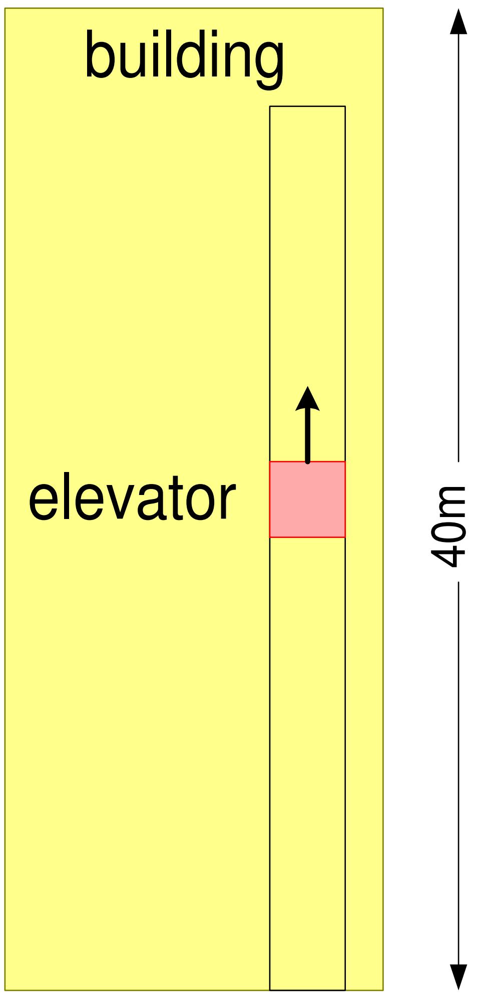
$t_{top\ floor}$ = time to reach top floor

v_{max} = maximum velocity

a_{max} = maximum acceleration

j_{max} = maximum jerk

Initial Estimates via Googling



Google "elevator" and "jerk":

$t_{top\ floor}$ ~= 16 s

v_{max} ~= 2.5 m/s

12% of gravity; weight goes up — a_{max} ~= 1.2 m/s^2 (up)

(v_{max} and a_{max}) relates to motor design and energy consumption

j_{max} ~= 2.5 m/s^3 — relates to control design

humans feel changes of forces
high jerk values are uncomfortable

numbers from: http://www.sensor123.com/vm_eva625.htm
CEP Instruments Pte Ltd Singapore

Exercise Time to Reach Top Floor Kinematic

input data

$S_0 = 0m$ $S_t = 40m$

$v_{max} = 2.5$ m/s

$a_{max} = 1.2$ m/s ^{2} (up)

$j_{max} = 2.5$ m/s ^{3}

elementary formulas

$$v = \frac{dS}{dt} \qquad a = \frac{dv}{dt} \qquad j = \frac{da}{dt}$$

Position in case of uniform acceleration:

$$S_t = S_0 + v_0 t + \frac{1}{2} a_0 t^2$$

exercises

Make a model for $t_{top\ floor}$

Make 0 ^{e} order model, based on constant velocity

Make 1 ^{e} order model, based on constant acceleration

What do you conclude from these models?

Models for Time to Reach Top Floor

input data

$S_0 = 0m$ $S_t = 40m$

$v_{max} = 2.5\ m/s$

$a_{max} = 1.2\ m/s^2$ (up)

$j_{max} = 2.5\ m/s^3$

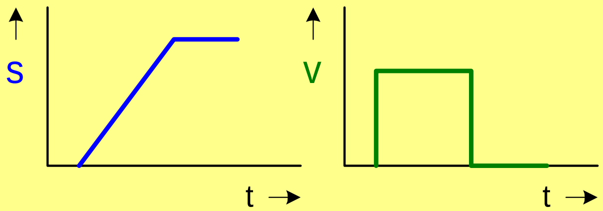
elementary formulas

$v = \frac{dS}{dt}$ $a = \frac{dv}{dt}$ $j = \frac{da}{dt}$

Position in case of uniform acceleration:

$S_t = S_0 + v_0 t + \frac{1}{2} a_0 t^2$

0^{th} order model

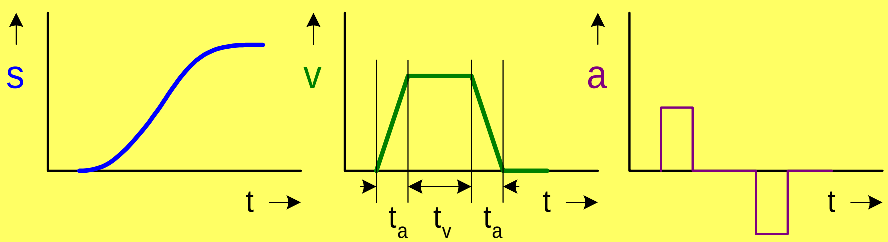


$S_{top\ floor} = v_{max} * t_{top\ floor}$

$40 = 2.5 * t_{top\ floor}$

$t_{top\ floor} = 40/2.5 =$ **16s**

1st order model



$t_{top\ floor} = t_a + t_v + t_a$

$t_a = v_{max} / a_{max}$

$S(t_a) = \frac{1}{2} * a_{max} * t_a^2$

$S_{linear} = S_{top\ floor} - 2 * S(t_a)$

$t_v = S_{linear} / v_{max}$

t_a ~= 2.5/1.2 ~= 2s

$S(t_a)$ ~= 0.5 * 1.2 * 2^2

$S(t_a)$ ~= 2.4m

t_v ~= (40-2*2.4) /2.5

t_v ~= 14s

$t_{top\ floor}$ ~= 2 + 14 + 2

$t_{top\ floor}$ ~= **18s**

Conclusions Move to Top Floor

Conclusions

v_{max} dominates traveling time

The model for the large height traveling time can be simplified into:

$$t_{travel} = S_{travel}/v_{max} + (t_a + t_j)$$

Exercise Elevator Performance

exercises

Make a model for t top floor

Take door opening and docking into account

What do you conclude from this model?

Elevator Performance Model

functional model

close doors → undock elevator → move elevator → dock elevator → open doors elevator

performance model

$$t_{top\ floor} = t_{close} + t_{undock} + t_{move} + t_{dock} + t_{open}$$

assumptions

$t_{close} \sim= t_{open} \sim= 2s$

$t_{undock} \sim= 1s$

$t_{dock} \sim= 2s$

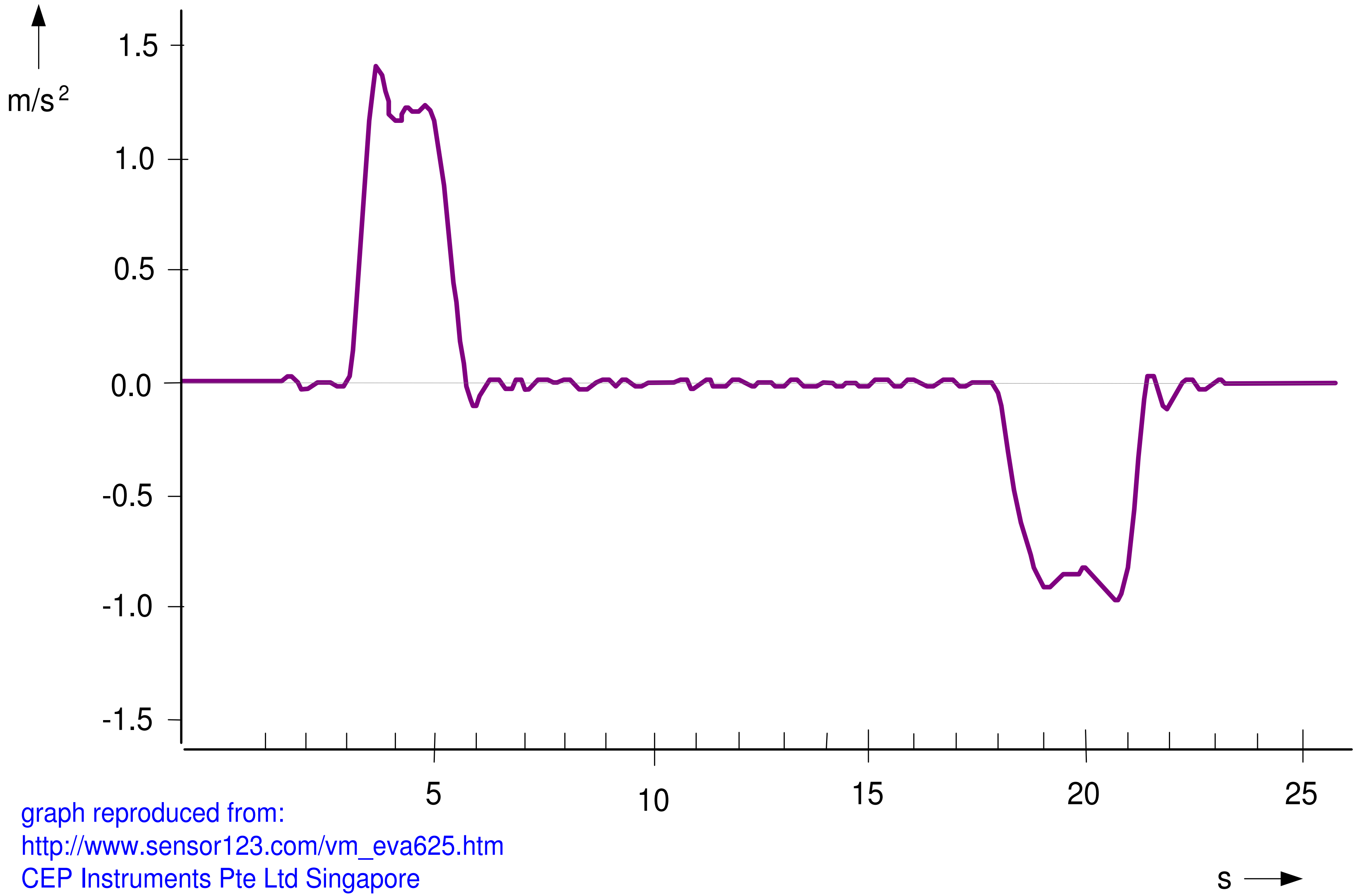
$t_{move} \sim= 18s$

outcome

$t_{top\ floor} \sim= 2 + 1 + 18 + 2 + 2$

$t_{top\ floor} \sim=$ **25s**

Measured Elevator Acceleration



graph reproduced from:
http://www.sensor123.com/vm_eva625.htm
CEP Instruments Pte Ltd Singapore

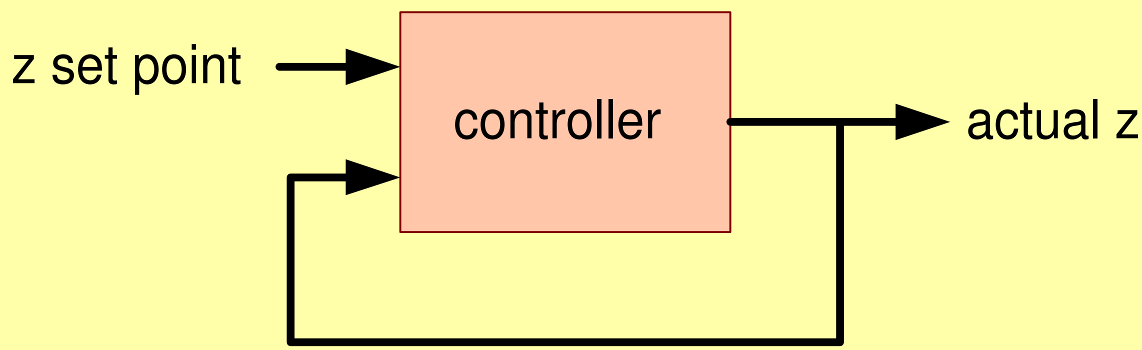
Theory versus Practice

What did we ignore or forget?

acceleration: up <> down 1.2 m/s^2 vs 1.0 m/s^2

slack, elasticity, damping et cetera of cables, motors....

controller impact



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Conclusions Performance Model Top Floor

Conclusions

The time to move is dominating the traveling time.

Docking and door handling is significant part of the traveling time.

$$t_{top\ floor} = t_{travel} + t_{elevator\ overhead}$$

Exercise Elevator Performance (2)

exercises

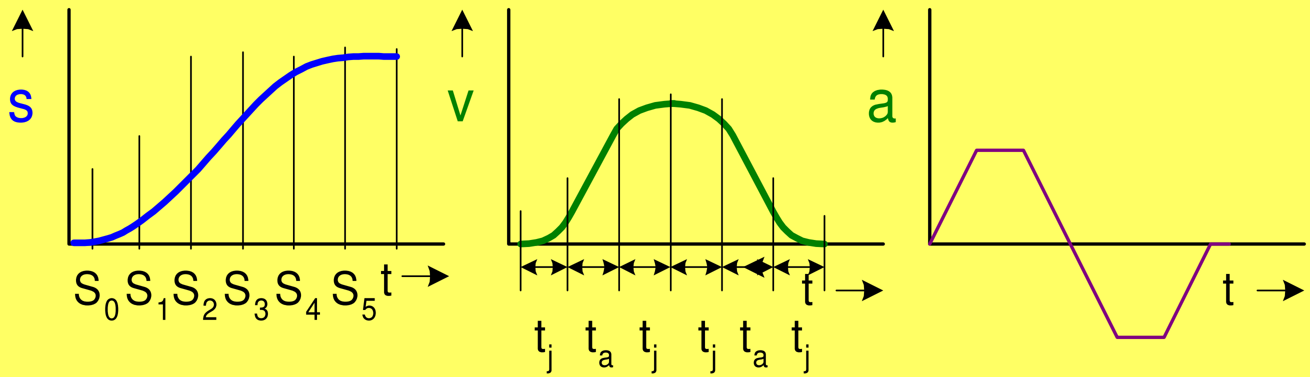
Make a model for t one floor

Take door opening and docking into account

What do you conclude from this model?

2nd Order Model Moving One Floor

2^{nd} order model



input data

S_0 = 0m S_t = 3m

v_{max} = 2.5 m/s

a_{max} = 1.2 m/s ^{2} (up)

j_{max} = 2.5 m/s ^{3}

$t_{one\ floor} = 2\ t_a + 4\ t_j$

$t_j = a_{max} / j_{max}$

$S_1 = 1/6 * j_{max}\ t_j^3$

$v_1 = 0.5\ j_{max}\ t_j^2$

$S_2 = S_1 + v_1 t_a + 0.5\ a_{max} t_a^2$

$v_2 = v_1 + a_{max}\ t_a$

$S_3 = S_2 + v_2 t_j + 0.5\ a_{max} t_j^2 - 1/6\ j_{max} t_j^3$

$S_3 = 0.5\ S_t$

t_j ~= 1.2/2.5 ~= 0.5s

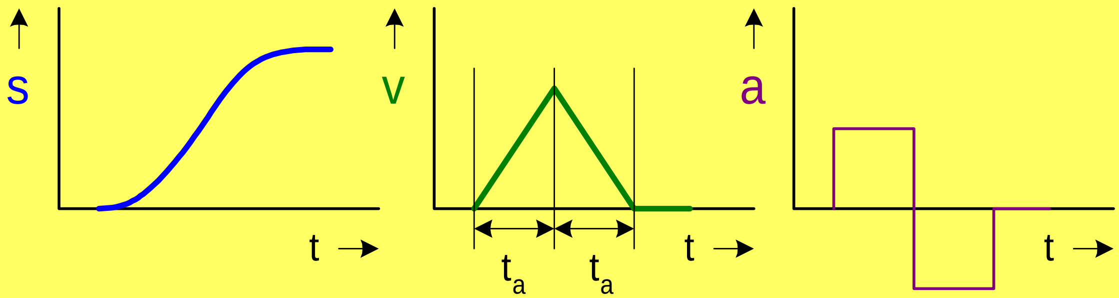
S_1 ~= 1/6 * 2.5 * 0.5 ^{3} ~= 0.05m

v_1 ~= 0.5 * 2.5 * 0.5 ^{2} ~= 0.3m/s

et cetera

1st Order Model Moving One Floor

1st order model



input data

S_0 = 0m S_t = 3m

v_{max} = 2.5 m/s

a_{max} = 1.2 m/s ^{2} (up)

j_{max} = 2.5 m/s ^{3}

$t_{one\ floor} = 2\ t_a$

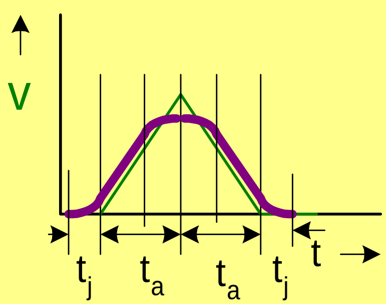
$S(t_a) = \frac{1}{2} * a_{max} * t_a^2 = 1.5m$

t_a ~= sqrt(1.5 / (0.5*1.2)) ~= 1.6s

$v(t_a)$ ~= $a_m t_a$ ~= 1.2*1.6 ~= 1.9 m/s

$t_{one\ floor}$ ~= **3s**

coarse 2nd order correction



$t_{one\ floor} = 2\ t_a + 2\ t_j$

t_j ~= 0.5s

$t_{one\ floor}$ ~= 2*1.6 + 2*0.5 ~= **4s**

Conclusions Move One Floor

Conclusions

a_{max} dominates travel time

The model for small height traveling time can be simplified into:

$$t_{travel} = 2\ sqrt(S_{travel}/a_{max}) + t_j$$

Elevator Performance Model

functional model

- close doors
- undock elevator
- move elevator
- dock elevator
- open doors elevator

performance model one floor (3m)

$$t_{\text{one floor}} = t_{\text{close}} + t_{\text{undock}} + t_{\text{move}} + t_{\text{dock}} + t_{\text{open}}$$

assumptions

$$t_{\text{close}} \sim= t_{\text{open}} \sim= 2s$$

$$t_{\text{undock}} \sim= 1s$$

$$t_{\text{dock}} \sim= 2s$$

$$t_{\text{move}} \sim= 4s$$

outcome

$$t_{\text{one floor}} \sim= 2 + 1 + 4 + 2 + 2$$

$$t_{\text{one floor}} \sim= \mathbf{11\ s}$$

Conclusions Performance Model One Floor

Conclusions

Overhead of docking and opening and closing doors is dominating traveling time.

Fast docking and fast door handling has significant impact on traveling time.

$$t_{\text{one floor}} = t_{\text{travel}} + t_{\text{elevator overhead}}$$

Exercise Time Line

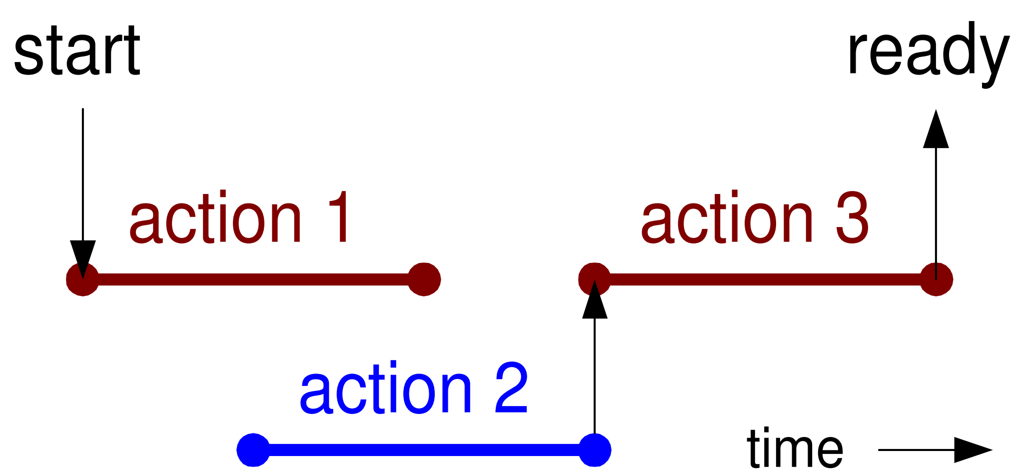
Exercise

Make a time line of people using the elevator.

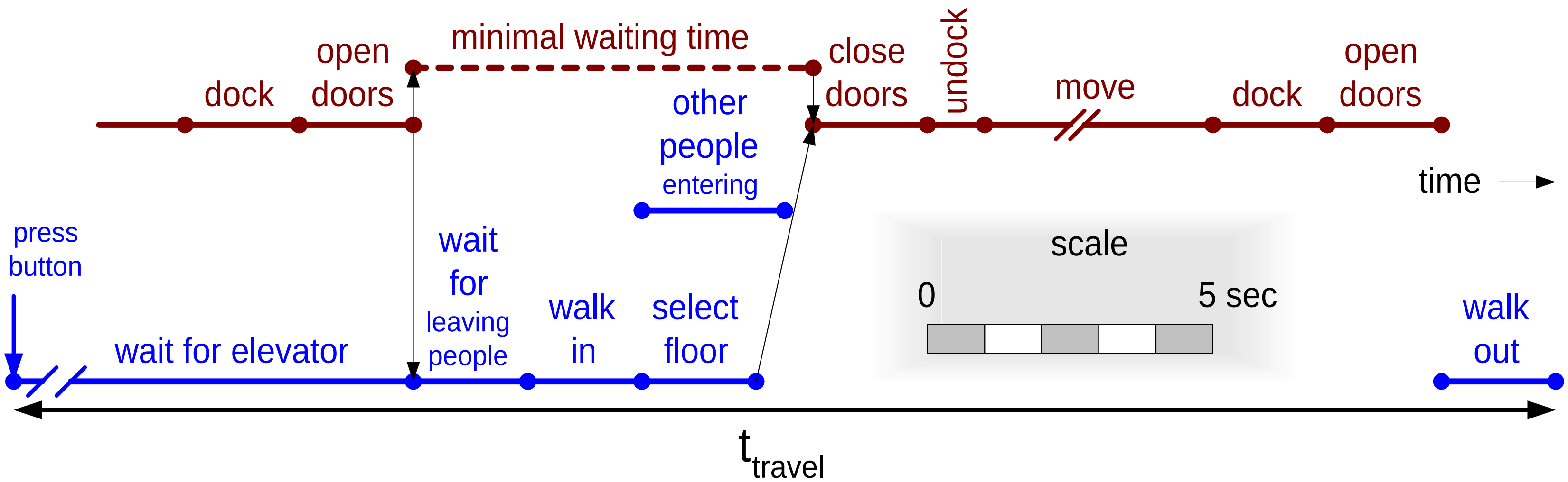
Estimate the time needed to travel to the top floor.

Estimate the time needed to travel one floor.

What do you conclude?



Time Line; Humans Using the Elevator



assumptions human dependent data

$t_{wait\ for\ elevator}$ = [0..2 minutes] depends heavily on use

$t_{wait\ for\ leaving\ people}$ = [0..20 seconds] idem

$t_{walk\ in}$ ~= 2 s

$t_{select\ floor}$ ~= 2 s

assumptions additional elevator data

$t_{minimal\ waiting\ time}$ ~= 8s

$t_{top\ floor}$ ~= 25s

$t_{one\ floor}$ ~= 11s

outcome

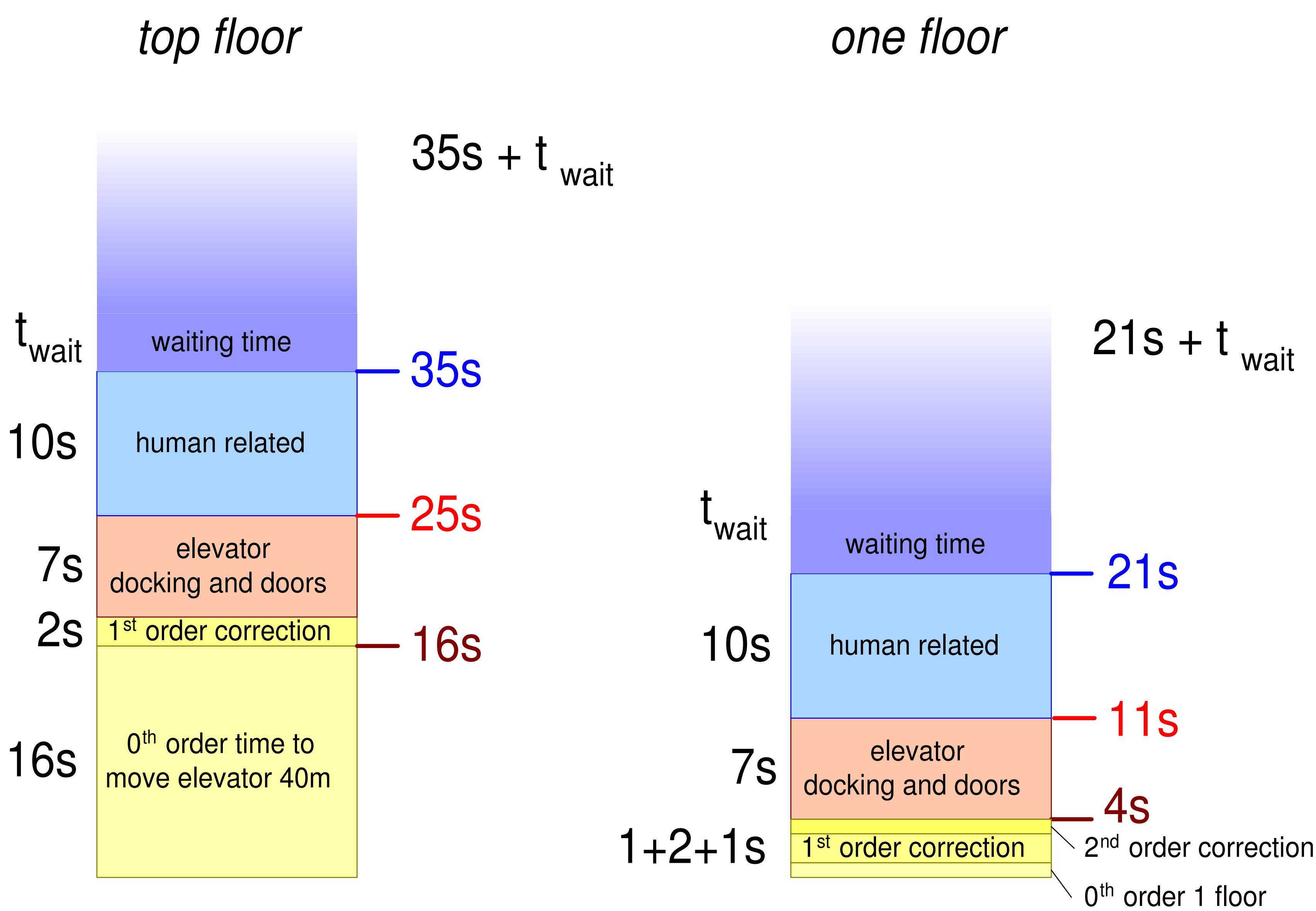
$t_{one\ floor}$ ~= 8 +2 + 11 + t_{wait}

~= **21 s** + t_{wait}

$t_{top\ floor}$ ~= 8 +2 + 25 + t_{wait}

~= **35 s**+ t_{wait}

Overview of Results for One Elevator



Conclusions Time Line

Conclusions

The human related activities have significant impact on the end-to-end time.

The waiting times have significant impact on the end-to-end time and may vary quite a lot.

$$t_{\text{end-to-end}} = t_{\text{human activities}} + t_{\text{wait}} + t_{\text{elevator travel}}$$

Exercise Energy and Power

Exercise

Estimate the energy consumption and the average and peak power needed to travel to the top floor.

What do you conclude?

Energy and Power Model

input data

S_0 = 0m

S_t = 40m

v_{max} = 2.5 m/s

$m_{elevator}$ = 1000 Kg (incl counter weight)

a_{max} = 1.2 m/s ^{2} (up)

$m_{passenger}$ = 100 Kg

j_{max} = 2.5 m/s ^{3}

1 passenger going up

g = 10 m/s ^{2}

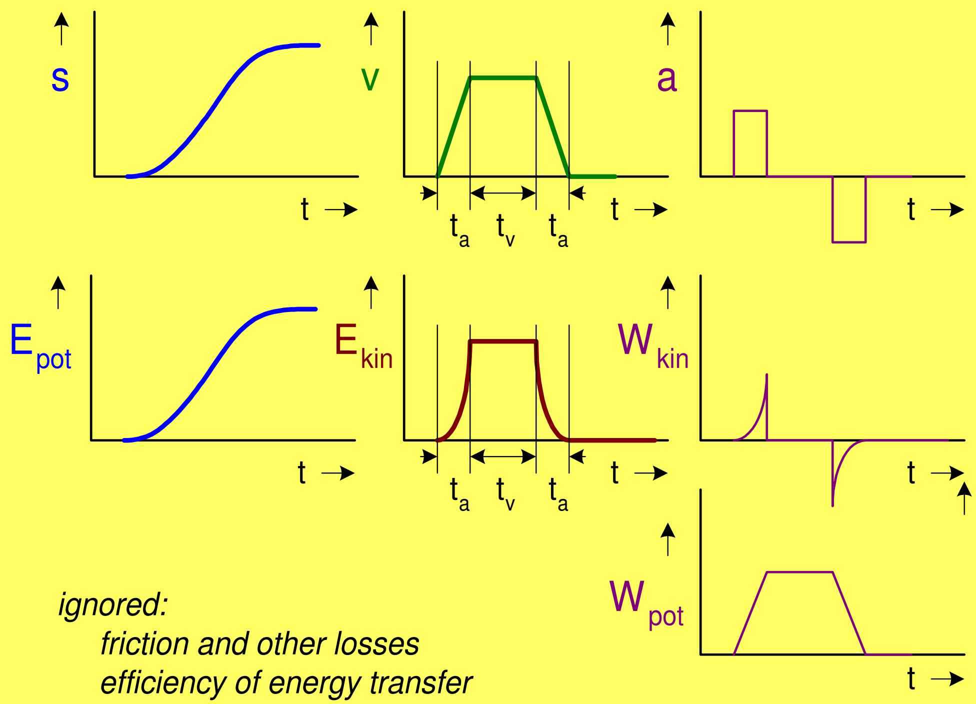
elementary formulas

$E_{kin} = 1/2\, mv^2$

$E_{pot} = mgh$

$$W = \frac{dE}{dt}$$

1st order model



ignored:
friction and other losses
efficiency of energy transfer

$E_{kin\ max} = 1/2\ m\ v_{max}^2$

$\sim= 0.5 * 1100 * 2.5^2$

$\sim=$ **3.4 kJ**

$W_{kin\ max} = m\ v_{max}\ a_{max}$

$\sim= 1100 * 2.5 * 1.2$

$\sim=$ **3.3 kW**

$E_{pot} = mgh$

$\sim= 100 * 10 * 40$

$\sim=$ **40 kJ**

$W_{pot\ max} \sim= E_{pot}/t_v$

$\sim= 40/16$

$\sim=$ **2.5 kW**

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EPMenergyAndPowerModel

Energy and Power Conclusions

Conclusions	
E_{pot} dominates energy balance W_{pot} is dominated by v_{max} W_{kin} causes peaks in power consumption and absorption W_{kin} is dominated by v_{max} and a_{max}	$E_{kin\ max} = 1/2\ m\ v_{max}^2$ $\sim= 0.5 * 1100 * 2.5^2$ $\sim=$ **3.4 kJ** $W_{kin\ max} = m\ v_{max}\ a_{max}$ $\sim= 1100 * 2.5 * 1.2$ $\sim=$ **3.3 kW** $E_{pot} = mgh$ $\sim= 100 * 10 * 40$ $\sim=$ **40 kJ** $W_{pot\ max} \sim= E_{pot}/t_v$ $\sim= 40/16$ $\sim=$ **2.5 kW**

Exercise Qualities and Design Considerations

Exercise

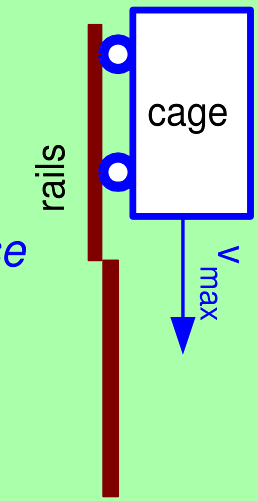
What other qualities and design considerations relate to the kinematic models?

Conclusions Qualities and Design Considerations

Examples of other qualities and design considerations

safety	v_{max}
acoustic noise	v_{max}, a_{max}, j_{max}
mechanical vibrations	v_{max}, a_{max}, j_{max}
air flow	?
operating life, maintenance	duty cycle, ?
...	

obstacles cause vibrations



Other Domains

applicability in other domains

kinematic modeling can be applied in a wide range of domains:

- transportation systems (trains, busses, cars, containers, ...)
- wafer stepper stages
- health care equipment patient handling
- material handling (printers, inserters, ...)
- MRI scanners gradient generation
- ...

Exercise Multiple Users

Exercise

Assume that a group of people enters the elevator at the ground floor. On every floor one person leaves the elevator.

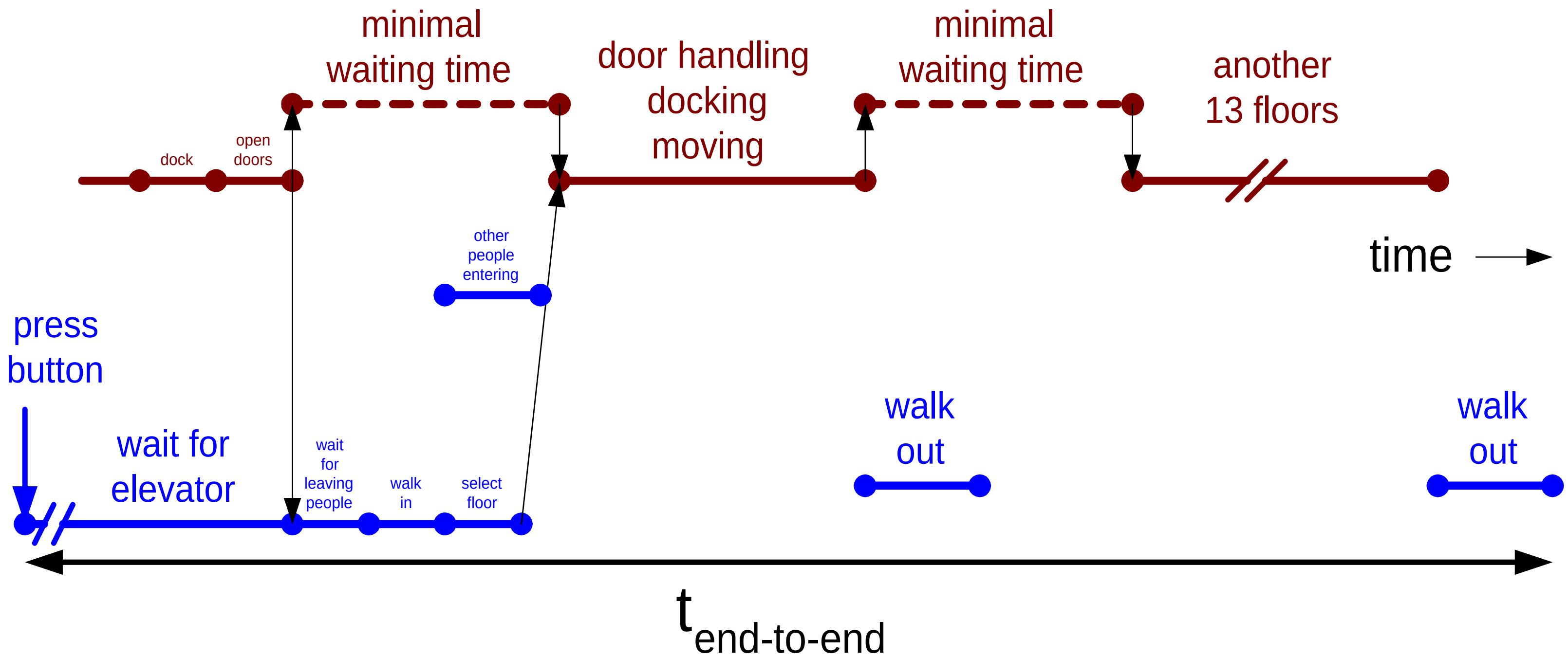
What is the end-to-end time for someone traveling to the top floor?

What is the desired end-to-end time?

What are potential solutions to achieve this?

What are the main parameters of the design space?

Multiple Users Model



elevator data

$t_{min\ wait}$ ~= 8s

$t_{one\ floor}$ ~= 11s

$t_{walk\ out}$ = 2s

n_{floors} = 40 div 3 +1 = 14

outcome

$t_{end\text{-}to\text{-}end}$ ~= 14 ($t_{min\ wait}$ + $t_{one\ floor}$) + $t_{walk\ out}$ + t_{wait}

~= 14 * (8 + 11) + 2 + t_{wait}

~= **268 s** + t_{wait}

$t_{non\text{-}stop}$ ~= 35 s + t_{wait}

Multiple Users Desired Performance

Considerations

desired time to travel to top floor ~< 1 minute

note that $t_{\text{wait next}} = t_{\text{travel up}} + t_{\text{travel down}}$

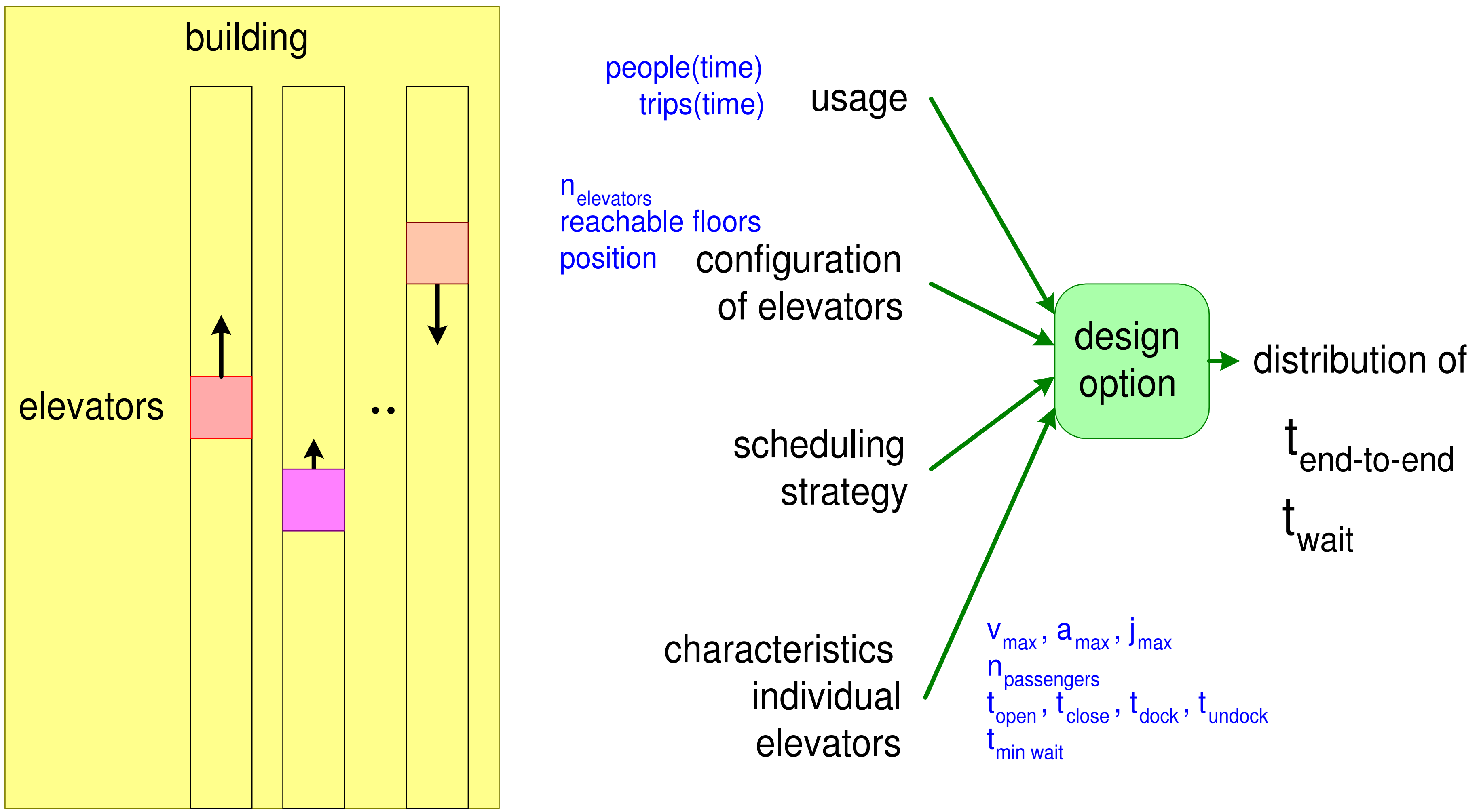
if someone just misses the elevator then the waiting time is

missed trip, return down, trip up

$t_{\text{end-to-end}} = 268 + 35 + 268 = 571\text{s} \sim= 10$ minutes!

desired waiting time ~< 1 minute

Design of Elevators System



Design of a system with multiple elevator requires a different kind of models: oriented towards logistics

Exceptional Cases

Exceptional Cases

non-functioning elevator

maintenance, cleaning of elevator

elevator used by people moving household

rush hour

special events (e.g. party, new years eve)

special floors (e.g. restaurant)

many elderly or handicapped people

playing children