Physical Models of an Elevator

by Gerrit Muller University of South-Eastern Norway-NISE

e-mail: gaudisite@gmail.com

www.gaudisite.nl

Abstract

An elevator is used as a simple system to model a few physical aspects. We will show simple kinematic models and we will consider energy consumption. These low level models are used to understand (physical) design considerations. Elsewhere we discuss higher level models, such as use cases and throughput, which complement these low level models.

\[
S_{\text{one floor}} = 3 \text{m} \\
v_{\text{max}} = 2.5 \text{ m/s} \\
a_{\text{max}} = 1.2 \text{ m/s}^2 \text{ (up)} \\
j_{\text{max}} = 2.5 \text{ m/s}^3 \\
S(t_a) = v(t_a) - \frac{1}{2} a_{\text{max}} t_a^2 \\
s_{\text{one floor}} \approx 2 \sqrt{\frac{S(t_a)}{0.5 a_{\text{max}}}} \\
v(t_a) \approx 1.2 \times 1.6 = 1.9 \text{ m/s} \\
2 a_{\text{max}} t_a = \sqrt{\frac{S(t_a)}{0.5 a_{\text{max}}}}
\]
Learning Goals

To understand the need for
- various views, e.g. physical, functional, performance
- mathematical models
- quantified understanding
- assumptions (when input data is unavailable yet) and later validation
- various visualizations, e.g. graphs
- understand and hence model at multiple levels of abstraction
- starting simple and expanding in detail, views, and solutions gradually, based on increased insight

To see the value and the limitations of these conceptual models

To appreciate the complementarity of conceptual models to other forms of modeling, e.g. problem specific models (e.g. structural or thermal analysis), SysML models, or simulations
warning

This presentation starts with a trivial problem.

Have patience!

Extensions to the trivial problem are used to illustrate many different modeling aspects.

Feedback on correctness and validity is appreciated
inhabitants want to reach their destination fast and comfortable

building owner and service operator have economic constraints:
space, cost, energy, ...

Physical Models of an Elevator

Gerrit Muller
Elementary Kinematic Formulas

\[ S_t = \text{position at time } t \]

\[ v_t = \text{velocity at time } t \]

\[ a_t = \text{acceleration at time } t \]

\[ j_t = \text{jerk at time } t \]

\[
 v = \frac{dS}{dt} \quad a = \frac{dv}{dt} \quad j = \frac{da}{dt}
\]

Position in case of uniform acceleration:

\[
 S_t = S_0 + v_0 t + \frac{1}{2} a_0 t^2
\]
Initial Expectations

What values do you expect or prefer for these quantities? Why?

\[ t_{\text{top floor}} = \text{time to reach top floor} \]
\[ v_{\text{max}} = \text{maximum velocity} \]
\[ a_{\text{max}} = \text{maximum acceleration} \]
\[ j_{\text{max}} = \text{maximum jerk} \]
Initial Estimates via Googling

Google "elevator" and "jerk":

- $v_{\text{max}} \approx 2.5 \text{ m/s}$
- $a_{\text{max}} \approx 1.2 \text{ m/s}^2$ (up)
- $j_{\text{max}} \approx 2.5 \text{ m/s}^3$
- $t_{\text{top floor}} \approx 16 \text{ s}$

12% of gravity; weight goes up

relates to motor design and energy consumption

relates to control design

humans feel changes of forces
high jerk values are uncomfortable

numbers from: http://www.sensor123.com/vm_eva625.htm
CEP Instruments Pte Ltd Singapore
**Exercise Time to Reach Top Floor Kinematic**

**input data**

- \( S_0 = 0 \text{m} \)
- \( S_t = 40 \text{m} \)
- \( v_{\text{max}} = 2.5 \text{ m/s} \)
- \( a_{\text{max}} = 1.2 \text{ m/s}^2 \) (up)
- \( j_{\text{max}} = 2.5 \text{ m/s}^3 \)

**elementary formulas**

\[
\begin{align*}
\text{Position in case of uniform acceleration:} \\
S_t &= S_0 + v_0 t + \frac{1}{2} a_0 t^2
\end{align*}
\]

**exercises**

- \( t_{\text{top floor}} \) is time needed to reach top floor without stopping
- Make a model for \( t_{\text{top floor}} \) and calculate its value
- Make 0\(^{\text{e}}\) order model, based on constant velocity
- Make 1\(^{\text{e}}\) order model, based on constant acceleration
- What do you conclude from these models?
**Models for Time to Reach Top Floor**

<table>
<thead>
<tr>
<th><strong>input data</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_0 = 0\text{m}$</td>
</tr>
<tr>
<td>$S_{\text{top floor}} = 40\text{m}$</td>
</tr>
<tr>
<td>$v_{\text{max}} = 2.5 \text{ m/s}$</td>
</tr>
<tr>
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<tr>
<td>$j_{\text{max}} = 2.5 \text{ m/s}^3$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>elementary formulas</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>$v = \frac{dS}{dt}$</td>
</tr>
<tr>
<td>$a = \frac{dv}{dt}$</td>
</tr>
<tr>
<td>$j = \frac{da}{dt}$</td>
</tr>
</tbody>
</table>

Position in case of uniform acceleration:

$S_t = S_0 + v_0t + \frac{1}{2}a_0t^2$

Position in case of uniform acceleration:

$S_{\text{linear}} = S_{\text{top floor}} - 2 \cdot S(t_a)$

$t_{\text{top floor}} = t_a + t_v + t_a$

$t_a = \frac{v_{\text{max}}}{a_{\text{max}}}$

$t_v = \frac{S_{\text{linear}}}{v_{\text{max}}}$

$t_{\text{top floor}} = 2 + 14 + 2$

$t_{\text{top floor}} = 18\text{s}$

$t_{\text{top floor}} = \frac{S_{\text{top floor}}}{v_{\text{max}}}$

$t_{\text{top floor}} = 16\text{s}$

$t_a \approx 2.5/1.2 \approx 2\text{s}$

$t_v \approx (40-2*2.4)/2.5$

$t_v \approx 14\text{s}$

$t_{\text{top floor}} \approx 18\text{s}$

$S(t_a) \approx 0.5 \cdot 1.2 \cdot 2^2$

$S(t_a) \approx 2.4\text{m}$

$S(t_a) = \frac{1}{2} \cdot a_{\text{max}} \cdot t_a^2$
Conclusions

\( v_{\text{max}} \) dominates traveling time

The model for the large height traveling time can be simplified into:

\[
\text{t}_{\text{travel}} = \frac{S_{\text{travel}}}{v_{\text{max}}} + (t_a + t_j)
\]
Exercise Time to Travel One Floor

**input data**

- $S_0 = 0 \text{m}$
- $S_{\text{top floor}} = 40 \text{m}$
- $v_{\text{max}} = 2.5 \text{ m/s}$
- $a_{\text{max}} = 1.2 \text{ m/s}^2$ (up)
- $j_{\text{max}} = 2.5 \text{ m/s}^3$

**elementary formulas**

\[
v = \frac{dS}{dt} \quad a = \frac{dv}{dt} \quad j = \frac{da}{dt}
\]

- Position in case of uniform acceleration:

\[
S_t = S_0 + v_0 t + \frac{1}{2} a_0 t^2
\]

**exercise**

Make a model for one floor and calculate it

What do you conclude from this model?
### 2nd Order Model Moving One Floor

#### 2nd order model

- **Input data**
  - $S_0 = 0$ m
  - $S_{one\ floor} = 3$ m
  - $v_{\text{max}} = 2.5$ m/s
  - $a_{\text{max}} = 1.2$ m/s$^2$ (up)
  - $j_{\text{max}} = 2.5$ m/s$^3$

- **Equations**
  - $t_{one\ floor} = 2 \ t_a + 4 \ t_j$
  - $t_j = a_{\text{max}} \ / \ j_{\text{max}}$
  - $S_1 = 1/6 \ * \ j_{\text{max}} t_j^3$
  - $v_1 = 0.5 \ j_{\text{max}} t_j^2$
  - $S_2 = S_1 + v_1 t_a + 0.5 \ a_{\text{max}} t_a^2$
  - $v_2 = v_1 + a_{\text{max}} t_a$
  - $S_3 = S_2 + v_2 t_j + 0.5 \ a_{\text{max}} t_j^2 - 1/6 \ j_{\text{max}} t_j^3$

- $S_3 = 0.5 \ S_1$
- $t_j \sim = 1.2/2.5 \sim = 0.5$ s
- $S_1 \sim = 1/6 * 2.5 * 0.5^3 \sim = 0.05$ m
- $v_1 \sim = 0.5 * 2.5 * 0.5^2 \sim = 0.3$ m/s

**et cetera**
**1st Order Model Moving One Floor**

![Diagram of 1st order model](image)

**Input Data**

- \( S_{0} = 0 \text{m} \)
- \( S_{\text{one floor}} = 3 \text{m} \)
- \( v_{\text{max}} = 2.5 \text{ m/s} \)
- \( a_{\text{max}} = 1.2 \text{ m/s}^2 \) (up)
- \( j_{\text{max}} = 2.5 \text{ m/s}^3 \)

**1st Order Model**

\[
S(t_a) = \frac{1}{2} \times a_{\text{max}} \times t_a^2
\]

\[
t_a = \sqrt{\frac{S(t_a)}{0.5 \times a_{\text{max}}}}
\]

\[
t_{\text{one floor}} = 2 \times t_a = 2 \sqrt{\frac{S(t_a)}{0.5 \times a_{\text{max}}}}
\]

\[
v(t_a) = a_{\text{max}} t_a \quad v(t_a) \approx 1.2 \times 1.6 \approx 1.9 \text{ m/s}
\]

**Coarse 2nd Order Correction**

\[
t_{\text{one floor}} \approx 2 \times t_a + 2 \times t_j
\]

\[
t_j \approx 0.5 \text{ s}
\]

\[
t_{\text{one floor}} \approx 2 \times 1.6 + 2 \times 0.5 \approx 4 \text{ s}
\]
Conclusions

$a_{\text{max}}$ dominates travel time

The model for small height traveling time can be simplified into:

$$t_{\text{travel}} = 2 \sqrt{S_{\text{travel}}/0.5 \ a_{\text{max}}} + t_j$$
exercise

Make a model for \( t_{\text{top floor}} \)

Take door opening and docking into account

What do you conclude from this model?
Elevator Performance Model

**functional model**
- close doors
- undock elevator
- move elevator
- dock elevator
- open doors elevator

**performance model**
\[ t_{\text{top floor}} = t_{\text{close}} + t_{\text{undock}} + t_{\text{move}} + t_{\text{dock}} + t_{\text{open}} \]

**assumptions**
- \( t_{\text{close}} \sim t_{\text{open}} \sim 2 \text{s} \)
- \( t_{\text{undock}} \sim 1 \text{s} \)
- \( t_{\text{dock}} \sim 2 \text{s} \)
- \( t_{\text{move}} \sim 18 \text{s} \)

**outcome**
- \( t_{\text{top floor}} \sim 2 + 1 + 18 + 2 + 2 \)
- \( t_{\text{top floor}} \sim 25 \text{s} \)
Conclusions

The time to move is dominating the traveling time.

Docking and door handling is significant part of the traveling time.

\[ t_{\text{top floor}} = t_{\text{travel}} + t_{\text{elevator overhead}} \]
What did we ignore or forget?

acceleration: up <> down $1.2 \text{ m/s}^2$ vs $1.0 \text{ m/s}^2$

slack, elasticity, damping et cetera of cables, motors....

ccontroller impact

......
exercise

Make a model for $t_{\text{one floor}}$

Take door opening and docking into account

What do you conclude from this model?
Elevator Performance Model

**functional model**

- close doors
- undock elevator
- move elevator
- dock elevator
- open doors elevator

**performance model one floor (3m)**

\[ t_{\text{one floor}} = t_{\text{close}} + t_{\text{undock}} + t_{\text{move}} + t_{\text{dock}} + t_{\text{open}} \]

**assumptions**

- \( t_{\text{close}} \sim= t_{\text{open}} \sim= 2 \text{s} \)
- \( t_{\text{undock}} \sim= 1 \text{s} \)
- \( t_{\text{dock}} \sim= 2 \text{s} \)
- \( t_{\text{move}} \sim= 4 \text{s} \)

**outcome**

- \( t_{\text{one floor}} \sim= 2 + 1 + 4 + 2 + 2 \)
- \( t_{\text{one floor}} \sim= 11 \text{s} \)
Conclusions

Overhead of docking and opening and closing doors is dominating traveling time.

Fast docking and fast door handling has significant impact on traveling time.

\[ t_{\text{one floor}} = t_{\text{travel}} + t_{\text{elevator overhead}} \]
Exercise

Make a time line of people using the elevator. Estimate the time needed to travel to the top floor. Estimate the time needed to travel one floor. What do you conclude?
Time Line; Humans Using the Elevator

Outcome

t_{one floor} \approx 8 + 2 + 11 + t_{wait}
\approx 21 \text{ s} + t_{wait}

t_{top floor} \approx 8 + 2 + 25 + t_{wait}
\approx 35 \text{ s} + t_{wait}

Assumptions Human Dependent Data

t_{wait for elevator} = [0..2 minutes] depends heavily on use

t_{wait for leaving people} = [0..20 seconds] idem

\[ t_{walk in} \approx t_{walk out} \approx 2 \text{ s} \]

\[ t_{select floor} \approx 2 \text{ s} \]

Assumptions Additional Elevator Data

\[ t_{minimal waiting time} \approx 8 \text{ s} \]

\[ t_{travel top floor} \approx 25 \text{ s} \]

\[ t_{travel one floor} \approx 11 \text{ s} \]
Overview of Results for One Elevator

**top floor**

- **0th order time to move elevator 40m**: 16s
- **1st order correction elevator docking and doors**: 2s
- **human related waiting time**: 7s
- **total waiting time**: t_wait = 35s + t_wait

**one floor**

- **0th order time to move elevator 40m**: 16s
- **1st order correction**: 1st order model
  - **elevator docking and doors**: 2s
  - **human related waiting time**: 10s
- **total waiting time**: t_wait = 21s + t_wait

---

Physical Models of an Elevator

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Conclusions

The human related activities have significant impact on the end-to-end time.

The waiting times have significant impact on the end-to-end time and may vary quite a lot.

\[ t_{\text{end-to-end}} = t_{\text{human activities}} + t_{\text{wait}} + t_{\text{elevator travel}} \]
Exercise

Estimate the energy consumption and the average and peak power needed to travel to the top floor.

What do you conclude?
## Energy and Power Model

**Input Data**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_0$</td>
<td>0 m</td>
</tr>
<tr>
<td>$S_t$</td>
<td>40 m</td>
</tr>
<tr>
<td>$v_{\text{max}}$</td>
<td>2.5 m/s</td>
</tr>
<tr>
<td>$a_{\text{max}}$</td>
<td>1.2 m/s² (up)</td>
</tr>
<tr>
<td>$\dot{v}_{\text{max}}$</td>
<td>2.5 m/s³</td>
</tr>
<tr>
<td>$g$</td>
<td>10 m/s²</td>
</tr>
</tbody>
</table>

**Elementary Formulas**

- $E_{\text{kin}} = \frac{1}{2} m v^2$
- $E_{\text{pot}} = mgh$
- $W = \frac{\text{d}E}{\text{d}t}$

**1st Order Model**

- $E_{\text{kin max}} = \frac{1}{2} m v_{\text{max}}^2$
  - $\approx 0.5 \times 1100 \times 2.5^2$
  - $\approx 3.4 \text{ kJ}$

- $W_{\text{kin max}} = m v_{\text{max}} a_{\text{max}}$
  - $\approx 1100 \times 2.5 \times 1.2$
  - $\approx 3.3 \text{ kW}$

- $E_{\text{pot}} = mgh$
  - $\approx 100 \times 10 \times 40$
  - $\approx 40 \text{ kJ}$

- $W_{\text{pot max}} \approx E_{\text{pot}}/t_v$
  - $\approx 40/16$
  - $\approx 2.5 \text{ kW}$

**Ignored:**
- Friction and other losses
- Efficiency of energy transfer

1 passenger going up
## Conclusions

- \( E_{\text{pot}} \) dominates energy balance
- \( W_{\text{pot}} \) is dominated by \( v_{\text{max}} \)
- \( W_{\text{kin}} \) causes peaks in power consumption and absorption
- \( W_{\text{kin}} \) is dominated by \( v_{\text{max}} \) and \( a_{\text{max}} \)

\[
\begin{align*}
E_{\text{kin max}} &= \frac{1}{2} m v_{\text{max}}^2 \\
&\approx 0.5 \times 1100 \times 2.5^2 \\
&\approx 3.4 \text{ kJ}
\end{align*}
\]

\[
\begin{align*}
W_{\text{kin max}} &= m v_{\text{max}} a_{\text{max}} \\
&\approx 1100 \times 2.5 \times 1.2 \\
&\approx 3.3 \text{ kW}
\end{align*}
\]

\[
\begin{align*}
E_{\text{pot}} &= mgh \\
&\approx 100 \times 10 \times 40 \\
&\approx 40 \text{ kJ}
\end{align*}
\]

\[
\begin{align*}
W_{\text{pot max}} &\approx E_{\text{pot}/t_v} \\
&\approx 40/16 \\
&\approx 2.5 \text{ kW}
\end{align*}
\]
Exercise Qualities and Design Considerations

Exercise

What other qualities and design considerations relate to the kinematic models?
## Conclusions Qualities and Design Considerations

### Examples of other qualities and design considerations

<table>
<thead>
<tr>
<th>Quality</th>
<th>Considerations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Safety</td>
<td>$v_{\text{max}}$</td>
</tr>
<tr>
<td>Acoustic Noise</td>
<td>$v_{\text{max}}, a_{\text{max}}, j_{\text{max}}$</td>
</tr>
<tr>
<td>Mechanical Vibrations</td>
<td>$v_{\text{max}}, a_{\text{max}}, j_{\text{max}}$</td>
</tr>
<tr>
<td>Air Flow</td>
<td>$?$</td>
</tr>
<tr>
<td>Operating Life, Maintenance</td>
<td>duty cycle, $?$</td>
</tr>
</tbody>
</table>

Obstacles cause vibrations when moving up and down the rails.
applicability in other domains

kinematic modeling can be applied in a wide range of domains:
transportation systems (trains, busses, cars, containers, ...)
wafer stepper stages
health care equipment patient handling
material handling (printers, inserters, ...)
MRI scanners gradient generation
...
Exercise

Assume that a group of people enters the elevator at the ground floor. On every floor one person leaves the elevator.

What is the end-to-end time for someone traveling to the top floor?

What is the desired end-to-end time?

What are potential solutions to achieve this?

What are the main parameters of the design space?
Multiple Users Model

outcome \approx 13 \times (8 + 11) + 2 + t_{\text{wait}}

door handling
docking
moving
walk
out

t_{\text{end-to-end}}

press
button

wait for
elevator

minimal waiting
time

walk
out

minimal waiting
time

another
13 floors

physical models of an elevator

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EPMmultipleUsersTimeLine
Considerations

desired time to travel to top floor $\sim< 1$ minute

note that $t_{\text{wait next}} = t_{\text{travel up}} + t_{\text{travel down}}$

if someone just misses the elevator then the waiting time is

$$t_{\text{end-to-end}} \sim = 249 + 35 + 249 = 533 \text{s} \sim = 9 \text{ minutes}!$$

desired waiting time $\sim< 1$ minute
Design of a system with multiple elevator requires a different kind of models: oriented towards logistics.
**Exceptional Cases**

- non-functioning elevator
- maintenance, cleaning of elevator
- elevator used by people moving household
- rush hour
- special events (e.g. party, new years eve)
- special floors (e.g. restaurant)
- many elderly or handicapped people
- playing children
Wrap-up Exercise

Make a list of all **visualizations** and **representations** that we used during the exercises
Summary of Visualizations and Representations

Physical Models of an Elevator

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EPMsummaryVisualizations

mathematical formulas
schematic graphs
measurement graph
quantification
timeline, concurrency

S_t = S_0 + v_0 t + \frac{1}{2} a_0 t^2

t_{\text{top floor}} = t_{\text{close}} + t_{\text{undock}} + t_{\text{move}} + t_{\text{dock}} + t_{\text{open}}

t_{\text{top floor}} \approx 2 + 1 + 18 + 2 + 2

t_{\text{top floor}} \approx 25s

functional

close doors

undock elevator

move elevator

dock elevator

open doors elevator

physical

building
top floor
elevator
cage
rails
40m

functional model

press button

wait for elevator

walk in

select floor

other people entering

press button

wait for elevator

walk in

select floor

functional
t_{\text{travel}}
t_{\text{wait}}
t_{\text{wait}}

waiting time

21s + t_{\text{wait}}

human related

elevator docking and doors

1st order model

2nd order correction

11s

4s

3+1s