Physical Models of an Elevator

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Abstract

An elevator is used as a simple system to model a few physical aspects. We will show simple kinematic models and we will consider energy consumption. These low level models are used to understand (physical) design considerations. Elsewhere we discuss higher level models, such as use cases and throughput, which complement these low level models.
Learning Goals

To understand the need for

- various views, e.g. physical, functional, performance
- mathematical models
- quantified understanding
- assumptions (when input data is unavailable yet) and later validation
- various visualizations, e.g. graphs
- understand and hence model at multiple levels of abstraction
- starting simple and expanding in detail, views, and solutions gradually, based on increased insight

To see the value and the limitations of these conceptual models

To appreciate the complementarity of conceptual models to other forms of modeling, e.g. problem specific models (e.g. structural or thermal analysis), SysML models, or simulations
warning

This presentation starts with a trivial problem.

Have patience!

Extensions to the trivial problem are used to illustrate many different modeling aspects.

Feedback on correctness and validity is appreciated
inhabitants want to reach their destination fast and comfortable

building owner and service operator have economic constraints: space, cost, energy, ...
Elementary Kinematic Formulas

\[ S_t = \text{position at time } t \]
\[ v_t = \text{velocity at time } t \]
\[ a_t = \text{acceleration at time } t \]
\[ j_t = \text{jerk at time } t \]

Velocity:
\[ v = \frac{dS}{dt} \]

Acceleration:
\[ a = \frac{dv}{dt} \]

Jerk:
\[ j = \frac{da}{dt} \]

Position in case of uniform acceleration:
\[ S_t = S_0 + v_0 t + \frac{1}{2} a_0 t^2 \]
Initial Expectations

What values do you expect or prefer for these quantities? Why?

\[ t_{\text{top floor}} = \text{time to reach top floor} \]
\[ v_{\text{max}} = \text{maximum velocity} \]
\[ a_{\text{max}} = \text{maximum acceleration} \]
\[ j_{\text{max}} = \text{maximum jerk} \]
Initial Estimates via Googling

Google "elevator" and "jerk":

- $t_{\text{top floor}} \sim 16 \text{ s}$
- $v_{\text{max}} \sim 2.5 \text{ m/s}$
- $a_{\text{max}} \sim 1.2 \text{ m/s}^2$ (up)
- $j_{\text{max}} \sim 2.5 \text{ m/s}^3$ (relates to control design)

12% of gravity; weight goes up

Humans feel changes of forces; high jerk values are uncomfortable

Numbers from:
- http://www.sensor123.com(vm_eva625.htm
- CEP Instruments Pte Ltd Singapore

Physical Models of an Elevator

Gerrit Muller

version: 0.4
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EPMinitialEstimates
### Exercise Time to Reach Top Floor Kinematic

#### input data
- \( S_0 = 0 \text{m} \)
- \( S_t = 40 \text{m} \)
- \( v_{\text{max}} = 2.5 \text{m/s} \)
- \( a_{\text{max}} = 1.2 \text{m/s}^2 \) (up)
- \( j_{\text{max}} = 2.5 \text{m/s}^3 \)

#### elementary formulas
- \( v = \frac{dS}{dt} \)
- \( a = \frac{dv}{dt} \)
- \( j = \frac{da}{dt} \)

Position in case of uniform acceleration:
\[
S_t = S_0 + v_0 t + \frac{1}{2} a_0 t^2
\]

#### exercises
- \( t_{\text{top floor}} \) is time needed to reach top floor without stopping
- Make a model for \( t_{\text{top floor}} \) and calculate its value
- Make 0\textsuperscript{e} order model, based on constant velocity
- Make 1\textsuperscript{e} order model, based on constant acceleration
- What do you conclude from these models?
**Models for Time to Reach Top Floor**

*input data*

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_0 )</td>
<td>0m</td>
</tr>
<tr>
<td>( S_{\text{top floor}} )</td>
<td>40m</td>
</tr>
<tr>
<td>( v_{\text{max}} )</td>
<td>2.5 m/s</td>
</tr>
<tr>
<td>( a_{\text{max}} )</td>
<td>1.2 m/s² (up)</td>
</tr>
<tr>
<td>( j_{\text{max}} )</td>
<td>2.5 m/s³</td>
</tr>
</tbody>
</table>

*elementary formulas*

\[
\begin{align*}
 v &= \frac{dS}{dt} \\
 a &= \frac{dv}{dt} \\
 j &= \frac{da}{dt}
\end{align*}
\]

Position in case of uniform acceleration:

\[
S_t = S_0 + v_0 t + \frac{1}{2} a_0 t^2
\]

---

**0th order model**

\[
\begin{align*}
S_{\text{top floor}} &= v_{\text{max}} \cdot t_{\text{top floor}} \\
t_{\text{top floor}} &= S_{\text{top floor}} / v_{\text{max}} \\
t_{\text{top floor}} &= 40 / 2.5 = 16\text{s}
\end{align*}
\]

**1st order model**

\[
\begin{align*}
t_a &\approx 2.5 / 1.2 \approx 2\text{s} \\
S(t_a) &\approx 0.5 \cdot 1.2 \cdot 2^2 \\
S(t_a) &\approx 2.4\text{m} \\
t_v &\approx (40 - 2 \cdot 2.4) / 2.5 \\
t_v &\approx 14\text{s} \\
t_{\text{top floor}} &\approx 2 + 14 + 2 \\
t_{\text{top floor}} &\approx 18\text{s}
\end{align*}
\]
Conclusions

$v_{\text{max}}$ dominates traveling time

The model for the large height traveling time can be simplified into:

$$t_{\text{travel}} = S_{\text{travel}}/v_{\text{max}} + (t_a + t_j)$$
Exercise Time to Travel One Floor

**input data**

- \( S_0 = 0 \text{m} \)
- \( S_{\text{top floor}} = 40 \text{m} \)
- \( v_{\text{max}} = 2.5 \text{ m/s} \)
- \( a_{\text{max}} = 1.2 \text{ m/s}^2 \) (up)
- \( j_{\text{max}} = 2.5 \text{ m/s}^3 \)

**elementary formulas**

\[
\begin{align*}
v &= \frac{dS}{dt} \\
a &= \frac{dv}{dt} \\

j &= \frac{da}{dt}
\end{align*}
\]

Position in case of uniform acceleration:

\[
S_t = S_0 + v_0 t + \frac{1}{2} a_0 t^2
\]

**exercise**

Make a model for \( t_{\text{one floor}} \) and calculate it
What do you conclude from this model?
**2nd Order Model Moving One Floor**

**Input Data**
- $S_0 = 0$ m
- $S_{\text{one floor}} = 3$ m
- $v_{\text{max}} = 2.5$ m/s
- $a_{\text{max}} = 1.2$ m/s$^2$ (up)
- $j_{\text{max}} = 2.5$ m/s$^3$

**Equations**
- $t_{\text{one floor}} = 2t_a + 4t_j$
- $t_j = \frac{a_{\text{max}}}{j_{\text{max}}}$
- $S_1 = \frac{1}{6}j_{\text{max}}t_j^3$
- $v_1 = 0.5j_{\text{max}}t_j^2$
- $S_2 = S_1 + v_1t_a + 0.5a_{\text{max}}t_a^2$
- $v_2 = v_1 + a_{\text{max}}t_a$
- $S_3 = S_2 + v_2t_j + 0.5a_{\text{max}}t_j^2 - \frac{1}{6}j_{\text{max}}t_j^3$
- $S_3 = 0.5S_t$
- $t_j \approx 1.2/2.5 \approx 0.5$ s
- $S_1 \approx 1/6 * 2.5 * 0.5^3 \approx 0.05$ m
- $v_1 \approx 0.5 * 2.5 * 0.5^2 \approx 0.3$ m/s

**Et cetera**
1st Order Model Moving One Floor

1st order model

\[ S(t_a) = \frac{1}{2} \cdot a_{\text{max}} \cdot t_a^2 \]
\[ t_a = \sqrt{\frac{S(t_a)}{0.5 \cdot a_{\text{max}}}} \]
\[ t_{\text{one floor}} = 2 \cdot t_a = 2 \sqrt{\frac{S(t_a)}{0.5 \cdot a_{\text{max}}}} \]
\[ v(t_a) = a_m t_a \quad v(t_a) \approx 1.2 \cdot 1.6 \approx 1.9 \, \text{m/s} \]

coarse 2nd order correction

\[ t_{\text{one floor}} = 2 \cdot t_a + 2 \cdot t_j \]
\[ t_j \approx 0.5 \, \text{s} \]
\[ t_{\text{one floor}} \approx 2 \cdot 1.6 + 2 \cdot 0.5 \approx 4 \, \text{s} \]

input data

\[ S_0 = 0 \, \text{m} \quad S_{\text{one floor}} = 3 \, \text{m} \]
\[ v_{\text{max}} = 2.5 \, \text{m/s} \]
\[ a_{\text{max}} = 1.2 \, \text{m/s}^2 \, \text{（up）} \]
\[ j_{\text{max}} = 2.5 \, \text{m/s}^3 \]
\[ t_{\text{one floor}} \approx = 2 \sqrt{\frac{1.5}{0.5 \cdot 1.2}} \approx = 2 \cdot 1.6 \approx = 3 \, \text{s} \]
Conclusions

$a_{\text{max}}$ dominates travel time

The model for small height traveling time can be simplified into:

$$t_{\text{travel}} = 2 \sqrt{S_{\text{travel}}/0.5 \ a_{\text{max}}} + t_j$$
Exercise Elevator Performance

exercise

Make a model for $t_{\text{top floor}}$
Take door opening and docking into account
What do you conclude from this model?
Elevator Performance Model

**functional model**
- close doors
- undock elevator
- move elevator
- dock elevator
- open doors elevator

**performance model**
\[ t_{\text{top floor}} = t_{\text{close}} + t_{\text{undock}} + t_{\text{move}} + t_{\text{dock}} + t_{\text{open}} \]

**assumptions**
- \( t_{\text{close}} \sim t_{\text{open}} \sim 2 \text{s} \)
- \( t_{\text{undock}} \sim 1 \text{s} \)
- \( t_{\text{dock}} \sim 2 \text{s} \)
- \( t_{\text{move}} \sim 18 \text{s} \)

**outcome**
- \( t_{\text{top floor}} \sim 2 + 1 + 18 + 2 + 2 \)
- \( t_{\text{top floor}} \sim 25 \text{s} \)
Conclusions

The time to move is dominating the traveling time.

Docking and door handling is significant part of the traveling time.

\[ t_{\text{top floor}} = t_{\text{travel}} + t_{\text{elevator overhead}} \]
Measured Elevator Acceleration

graph reproduced from:
http://www.sensor123.com/vm_eva625.htm
CEP Instruments Pte Ltd Singapore
What did we ignore or forget?

acceleration: up <> down 1.2 m/s² vs 1.0 m/s²

slack, elasticity, damping et cetera of cables, motors....

ccontroller impact

.....
Exercise Time to Travel One Floor

**exercise**

Make a model for \( t_{\text{one floor}} \)

Take door opening and docking into account

What do you conclude from this model?
Elevator Performance Model

**functional model**

- Close doors
- Undock elevator
- Move elevator
- Dock elevator
- Open doors elevator

**performance model one floor (3m)**

\[ t_{\text{one floor}} = t_{\text{close}} + t_{\text{undock}} + t_{\text{move}} + t_{\text{dock}} + t_{\text{open}} \]

**assumptions**

- \( t_{\text{close}} \sim= t_{\text{open}} \sim= 2 \text{s} \)
- \( t_{\text{undock}} \sim= 1 \text{s} \)
- \( t_{\text{dock}} \sim= 2 \text{s} \)
- \( t_{\text{move}} \sim= 4 \text{s} \)

**outcome**

\[ t_{\text{one floor}} \sim = 2 + 1 + 4 + 2 + 2 \]

\[ t_{\text{one floor}} \sim = 11 \text{ s} \]
Conclusions

Overhead of docking and opening and closing doors is dominating traveling time.

Fast docking and fast door handling has significant impact on traveling time.

\[ t_{\text{one floor}} = t_{\text{travel}} + t_{\text{elevator overhead}} \]
Exercise

Make a time line of people using the elevator. Estimate the time needed to travel to the top floor. Estimate the time needed to travel one floor. What do you conclude?
assumptions human dependent data

\[ t_{\text{wait for elevator}} = [0..2 \text{ minutes}] \text{ depends heavily on use} \]
\[ t_{\text{wait for leaving people}} = [0..20 \text{ seconds}] \text{ idem} \]
\[ t_{\text{walk in}} \approx t_{\text{walk out}} \approx 2 \text{ s} \]
\[ t_{\text{select floor}} \approx 2 \text{ s} \]

outcome

\[ t_{\text{one floor}} = t_{\text{minimal waiting time}} + t_{\text{walk out}} + t_{\text{travel one floor}} + t_{\text{wait}} \]
\[ t_{\text{top floor}} = t_{\text{minimal waiting time}} + t_{\text{walk out}} + t_{\text{travel top floor}} + t_{\text{wait}} \]
\[ t_{\text{one floor}} \approx 8 + 2 + 11 + t_{\text{wait}} \]
\[ \approx 21 \text{ s} + t_{\text{wait}} \]
\[ t_{\text{top floor}} \approx 8 + 2 + 25 + t_{\text{wait}} \]
\[ \approx 35 \text{ s} + t_{\text{wait}} \]

assumptions additional elevator data

\[ t_{\text{minimal waiting time}} \approx 8 \text{ s} \]
\[ t_{\text{travel top floor}} \approx 25 \text{ s} \]
\[ t_{\text{travel one floor}} \approx 11 \text{ s} \]
Overview of Results for One Elevator

**top floor**

- $t_{\text{wait}}$: waiting time
- 10s: human related
- 7s: elevator docking and doors
- 2s: 1st order correction
- 16s: 0th order time to move elevator 40m

$35s + t_{\text{wait}}$

**one floor**

- $t_{\text{wait}}$: waiting time
- 21s: 2nd order correction
- 10s: human related
- 7s: elevator docking and doors
- 4s: 2nd order correction
- 3+1s: 1st order model

$21s + t_{\text{wait}}$
Conclusions

The human related activities have significant impact on the end-to-end time.

The waiting times have significant impact on the end-to-end time and may vary quite a lot.

\[ t_{\text{end-to-end}} = t_{\text{human activities}} + t_{\text{wait}} + t_{\text{elevator travel}} \]
Exercise

Estimate the energy consumption and the average and peak power needed to travel to the top floor.

What do you conclude?
**Energy and Power Model**

### Input Data
- \( S_0 = 0 \text{m} \)
- \( v_{\text{max}} = 2.5 \text{ m/s} \)
- \( a_{\text{max}} = 1.2 \text{ m/s}^2 \) (up)
- \( j_{\text{max}} = 2.5 \text{ m/s}^3 \)
- \( g = 10 \text{ m/s}^2 \)
- \( S_t = 40 \text{m} \)
- \( m_{\text{elevator}} = 1000 \text{ Kg (incl counter weight)} \)
- \( m_{\text{passenger}} = 100 \text{ Kg} \)

### Elementary Formulas
- \( E_{\text{kin}} = \frac{1}{2} m v^2 \)
- \( E_{\text{pot}} = mgh \)
- \( W = \frac{dE}{dt} \)

### 1st Order Model
- \( E_{\text{kin max}} = \frac{1}{2} m v_{\text{max}}^2 \)
  \(\approx 0.5 \times 1100 \times 2.5^2 \)
  \(\approx 3.4 \text{ kJ} \)
- \( W_{\text{kin max}} = m v_{\text{max}} a_{\text{max}} \)
  \(\approx 1100 \times 2.5 \times 1.2 \)
  \(\approx 3.3 \text{ kW} \)
- \( E_{\text{pot}} = mgh \)
  \(\approx 100 \times 10 \times 40 \)
  \(\approx 40 \text{ kJ} \)
- \( W_{\text{pot max}} \approx E_{\text{pot}}/t_v \)
  \(\approx 40/16 \)
  \(\approx 2.5 \text{ kW} \)

**ignored:**
- friction and other losses
- efficiency of energy transfer

1 passenger going up

1st order model

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EPMenergyAndPowerModel
**Conclusions**

- $E_{\text{pot}}$ dominates energy balance
- $W_{\text{pot}}$ is dominated by $v_{\text{max}}$
- $W_{\text{kin}}$ causes peaks in power consumption and absorption
- $W_{\text{kin}}$ is dominated by $v_{\text{max}}$ and $a_{\text{max}}$

\[
E_{\text{kin\ max}} = \frac{1}{2} m v_{\text{max}}^2 \\
~ = 0.5 \times 1100 \times 2.5^2 \\
~ = 3.4 \text{ kJ}
\]

\[
W_{\text{kin\ max}} = m v_{\text{max}} a_{\text{max}} \\
~ = 1100 \times 2.5 \times 1.2 \\
~ = 3.3 \text{ kW}
\]

\[
E_{\text{pot}} = mgh \\
~ = 100 \times 10 \times 40 \\
~ = 40 \text{ kJ}
\]

\[
W_{\text{pot\ max}} \approx \frac{E_{\text{pot}}}{t_v} \\
~ = \frac{40}{16} \\
~ = 2.5 \text{ kW}
\]
Exercise Qualities and Design Considerations

Exercise

What other qualities and design considerations relate to the kinematic models?
Examples of other qualities and design considerations

- Safety: $v_{\text{max}}$
- Acoustic noise: $v_{\text{max}}, a_{\text{max}}, j_{\text{max}}$
- Mechanical vibrations: $v_{\text{max}}, a_{\text{max}}, j_{\text{max}}$
- Air flow: ?
- Operating life, maintenance: duty cycle, ?

Obstacles cause vibrations
applicability in other domains

kinematic modeling can be applied in a wide range of domains:
- transportation systems (trains, busses, cars, containers, ...)
- wafer stepper stages
- health care equipment patient handling
- material handling (printers, inserters, ...)
- MRI scanners gradient generation

...
Exercise

Assume that a group of people enters the elevator at the ground floor. On every floor one person leaves the elevator.

What is the end-to-end time for someone traveling to the top floor?

What is the desired end-to-end time?

What are potential solutions to achieve this?

What are the main parameters of the design space?
Multiple Users Model

Outcome

\[ \text{outcome} \approx 13 \times (8 + 11) + 2 + t_{\text{wait}} \]

Elevator data

- \( t_{\text{min wait}} \approx 8 \text{s} \)
- \( t_{\text{one floor}} \approx 11 \text{s} \)
- \( t_{\text{walk out}} \approx 2 \text{s} \)
- \( n_{\text{floors}} = 40 \text{ div } 3 + 1 = 14 \)
- \( n_{\text{stops}} = n_{\text{floors}} - 1 = 13 \)

End-to-end time

\[ t_{\text{end-to-end}} = n_{\text{stops}} \left( t_{\text{min wait}} + t_{\text{one floor}} \right) + t_{\text{walk out}} + t_{\text{wait}} \]

\[ \approx 13 \times (8 + 11) + 2 + t_{\text{wait}} \]

\[ \approx 249 \text{ s} + t_{\text{wait}} \]

Non-stop time

\[ t_{\text{non-stop}} \approx 35 \text{ s} + t_{\text{wait}} \]
Considerations

desired time to travel to top floor $\sim< 1$ minute

note that $t_{\text{wait next}} = t_{\text{travel up}} + t_{\text{travel down}}$

if someone just misses the elevator then the waiting time is

\[
t_{\text{end-to-end}} \sim = 249 + 35 + 249 = 533s \sim = 9 \text{ minutes!}
\]

desired waiting time $\sim< 1$ minute
Design of Elevators System

Design of a system with multiple elevator requires a different kind of models: oriented towards logistics
Exceptional Cases

- non-functioning elevator
- maintenance, cleaning of elevator
- elevator used by people moving household
- rush hour
- special events (e.g. party, new years eve)
- special floors (e.g. restaurant)
- many elderly or handicapped people
- playing children
Wrap-up Exercise

Make a list of all *visualizations* and *representations* that we used during the exercises.
Physical Models of an Elevator

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EPMsummary/Visualizations

Summary of Visualizations and Representations

mathematical formulas
schematic graphs
measurement graph
quantification
timeline, concurrency

functional model
- close doors
- undock elevator
- move elevator
- dock elevator
- open doors elevator

functional

physical

building

top floor

elevator

t 40 m

rails

cage


top floor

mathematical formulas

$S_t = S_0 + v_0 t + \frac{1}{2} a_\theta t^2$

t_{top floor} = t_{close} + t_{undock} + t_{move} + t_{dock} + t_{open}$

$\text{t}_{\text{top floor}} \approx 2 + 1 + 18 + 2 + 2$

$\text{t}_{\text{top floor}} \approx 25$s

$\text{t}_{\text{wait}} \approx 21$s + $t_{\text{wait}}$

$\text{10}s$

human related

docking and doors

$\text{7}s$

$\text{11}s$

$\text{4}s$

$\text{2}s$

$\text{1}s$

$\text{3} + \text{1}s$

$\text{1}^{st}$ order model

$\text{2}^{nd}$ order correction

wait for elevator

walk in

other people entering

close doors

undock

move
dock

open doors

press button

wait for elevator

wait for leaving people

select floor

scale

0

5 sec

walk out

0

5

10

15

20

25

m/s²

m/s²

graph reproduced from:
http://www.sensor123.com/vm_eva625.htm

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-1.5

-1.0

-0.5

0.0

0.5

1.0

1.5

5

10

15

20

25

s

-1.5

-1.0

-0.5

0.0

0.5

1.0

1.5

5

10

15

20

25

s

graph reproduced from:
http://www.sensor123.com/vm_eva625.htm

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wait for elevator

walk in

select floor